Questions

Q1.

A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.

(1)

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,
(ii) there are at least 4 red beads in Aliya's bracelet.

(3)

(c) Comment on the suitability of a binomial distribution to model this situation.

(1)

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

(4)

(e) Find the *p*-value in this case.

(1)

(Total for question = 10 marks)

Q2.

In an experiment a group of children each repeatedly throw a dart at a target.
For each child, the random variable H represents the number of times the dart hits the target in the first 10 throws.

Peta models H as B(10, 0.1)

(a) State two assumptions Peta needs to make to use her model.

(2)

(b) Using Peta's model, find P($H \geq 4$)

(1)

For each child the random variable F represents the number of the throw on which the dart first hits the target.

Using Peta's assumptions about this experiment,

(c) find P($F = 5$)

(2)

Thomas assumes that in this experiment no child will need more than 10 throws for the dart to hit the target for the first time. He models P($F = n$) as

$$P(F = n) = 0.01 + (n - 1) \times \alpha$$

where α is a constant.

(d) Find the value of α

(4)

(e) Using Thomas' model, find P($F = 5$)

(1)

(f) Explain how Peta's and Thomas' models differ in describing the probability that a dart hits the target in this experiment.

(1)

(Total for question = 11 marks)

Q3.

Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(a) Find the probability that it has a daily mean total cloud cover of 6 or greater.

(1)

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim B(8, 0.76)$

Using Magali's model,

(b) (i) find $P(X \geq 6)$

(2)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

(2)

(c) Explain whether or not your answers to part (b) support the use of Magali's model.

(1)

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

(e) Comment on Magali's model in light of your answer to part (d).

(2)

(Total for question = 9 marks)

Q4.

(a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club.

A random sample of 36 students is taken from the university.

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$
(ii) $P(X \geq 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

(1)

A random sample of 50 students is taken from the university.

(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

(Total for question = 7 marks)

Mark Scheme

Q1.

Qu	Scheme	Marks	AO
(a)	[R = no. of red beads in Aliya's bracelet] $R \sim B(18, 0.14)$	B1 (1)	3.3
(b)(i)	$P(R = 1) = 0.19403\ldots$ awrt **0.194**	B1	1.1b
(ii)	$P(R \geqslant 4) = 1 - P(R \leqslant 3) = 1 - [0.76184\ldots]$	M1	3.4
	$= 0.2381588\ldots$ awrt **0.238**	A1 (3)	1.1b
(c)	Requires $p = 0.14$ to be constant so need a large number of beads in the sack to ensure that removing 18 beads does not appreciably affect this probability, then it could be suitable.	B1 (1)	3.5b
(d)	$H_0 : p = 0.14$ $H_1 : p \neq 0.14$	B1	2.5
	[X = number of red beads in the sample] $X \sim B(75, 0.14)$	M1	3.3
	$P(X \leqslant 4) = 0.01506\ldots$ or if B(75, 0.14) seen awrt 0.02	A1	3.4
	{0.02 < 0.025 so significant or reject H_0 } There is evidence that the proportion of red beads has changed	A1 (4)	2.2b
(e)	p-value is 2×"0.01506..." = 0.030123... = awrt 0.03	B1ft (1)	1.1b
		(10 marks)	

	Notes
(a)	B1 for B(18, 0.14) accept in words e.g. binomial with $n = 18$ and $p = 0.14$
(b)(i)	B1 for awrt 0.194
(ii)	M1 for interpreting "at least 4" Need $1 - P(R \leqslant 3)$ and $1 - p$ $[0 < p < 1]$ $P(R = 3) = 0.233..$ OK
	A1 for awrt 0.238
(c)	B1 for mention of large number of beads and need for $p = 0.14$ to be constant for it to be suitable. Do NOT accept e.g. "events are independent"
(d)	B1 for both hypotheses correct with use of p or π
	M1 for selecting a suitable model: sight or correct use of B(75, 0.14) May be implied by sight of 0.015 or better or $[P(X > 4) =]$ 0.9849… i.e. 0.985 or better
	1st A1 for use of the correct model awrt 0.015 (accept awrt 0.02 following a correct expression) Allow 1st A1 for awrt 0.985 only if correct comparison with 0.975 is seen. Sight of B(75, 0.14) and $P(X \leqslant 4)$ = awrt 0.02 scores M1A1
	No sight of B(75, 0.14) but sight of awrt 0.015 scores M1(⇒)A1[Condone $P(X = 4) = ..$]
	2nd A1 **(dep on M1A1)** for a correct conclusion in context mentioning "proportion", "red" and "changed"
	If there is a statement about H_0 or significance it must be compatible.
NB	May see CR i.e. $X \leqslant 4$ (mark when prob seen) and $X \geqslant 18$ (prob = 0.01406..) Ignore upper limit
	NB for information $P(X = 4) = 0.0104\ldots$ and can only score M1A0A0 if B(75, 0.14) seen
(e)	B1ft for awrt 0.03 Allow ft of their probability in (d) provided at least 3sf used
	NB an answer of 0.02 in (d) leading to 0.04 in (e) is B0
SC	Use of CR will give significance level of 0.01506…+ 0.01406… = 0.029… score B1 **no ft**

Q2.

Qu	Scheme	Marks	AO
(a)	The <u>probability</u> of a dart hitting the target is <u>constant</u> (from child to child and for each throw by each child) (o.e.) The <u>throws</u> of each of the darts are <u>independent</u> (o.e.)	B1 B1 (2)	1.2 1.2
(b)	[P($H \geqslant 4$) = 1 – P($H \leqslant 3$) = 1 – 0.9872 = 0.012795.. =] awrt **<u>0.0128</u>**	B1 (1)	1.1b
(c)	P($F = 5$) = $0.9^4 \times 0.1$, = 0.06561 = awrt **<u>0.0656</u>**	M1, A1 (2)	3.4 1.1b
(d)	<table><tr><td>n</td><td>1</td><td>2</td><td>…</td><td>10</td></tr><tr><td>P($F = n$)</td><td>0.01</td><td>$0.01 + \alpha$</td><td>…</td><td>$0.01+9\alpha$</td></tr></table> Sum of probs = 1 $\Rightarrow \frac{10}{2}[2\times 0.01+9\alpha]=1$ [i.e. $5(0.02 + 9\alpha) = 1$ or $0.1 + 45\alpha = 1$] so $\alpha =$ **<u>0.02</u>**	M1 M1A1 A1 (4)	3.1b 3.1a 1.1b 1.1b
(e)	P($F = 5$| Thomas' model) = **<u>0.09</u>**	B1ft (1)	3.4
(f)	<u>Peta's</u> model assumes the <u>probability</u> of hitting target is <u>constant</u> (o.e.) **and** <u>Thomas'</u> model assumes this <u>probability increases</u> with each attempt(o.e.)	B1 (1)	3.5a
		(11 marks)	

	Notes
(a)	1st B1 for stating that the <u>probability</u> (or possibility or chance) is <u>constant</u> (or fixed or same) 2nd B1 for stating that <u>throws</u> are <u>independent</u> ["trials" are independent is B0]
(b)	B1 for awrt 0.0128 (found on calculator)
(c)	M1 for a probability expression of the form $(1-p)^4 \times p$ where $0 < p < 1$ A1 for awrt 0.0656 SC Allow M1A0 for answer only of 0.066
(d)	1st M1 for setting up the distribution of F with at least 3 correct values of n and P($F = n$) in terms of α. (Can be implied by 2nd M1 or 1st A1) 2nd M1 for use of sum of probs = 1 **and** clear summation or use of arithmetic series formula (allow 1 error or missing term). (Can be implied by 1st A1) 1st A1 for a correct equation for α 2nd A1 for $\alpha = 0.02$ (must be exact and come from correct working)
(e)	B1ft for value resulting from $0.01 + 4\times$"their α" (provided α and the answer are probs) **Beware** If their answer is the same as their (c) (or a rounded version of their (c)) score B0
(f) ALT	B1 for a suitable comment about the <u>probability</u> of hitting the target Allow idea that Peta's model suggests the dart may never hit the target but Thomas' says that it will hit at least once (in the first 10 throws).

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{523+52+28}{184} = \frac{132}{184} = 0.717$	B1	This mark is given for a correct value for the probability for the cloud cover
(b)(i)	$P(X \geq 6) = 1 - P(X \leq 5)$	M1	This mark is given for using $1 - P(X \leq 5)$ with B(8, 0.76)
	$= 1 - 0.2967$ $= 0.703$	A1	This mark is given for finding as correct value for the probability
(b)(ii)	$184 \times P(X = 7)$ $= 184 \times 0.2811$	M1	This mark is given for using $184 \times P(X = 7)$ with B(8, 0.76)
	$= 51.7$	A1	This mark is given for finding as correct value for the probability
(c)	The answer to part (b)(i) of 0.703 is similar to 0.7127 in part (a) The answer to part (b)(ii) of 51.7 is very close to 52 found in the data set	B1	This mark is given for a correct evaluation of the outcomes from part (b) to determine the appropriateness of Magali's model
(d)	$\frac{5+9+9}{28} = \frac{23}{28} = 0.821$	B1	This mark is given for a correct value for the probability for the cloud cover
(e)	The answer to part (d) of 0.821 is greater than that in part (a) of 0.717 This shows that there is a higher chance of having high cloud cover if the previous day had high cloud cover	B1	This mark is given for a correct comparison for the answer to part (d) with the data set
	Thus independence does not hold so a binomial model might not be suitable	B1	This mark is given for a correct conclusion stated
			(Total 9 marks)

Q4.

	Scheme	Marks	AO
(a)	**Disadvantage:** e.g. Not random; cannot use (reliably) for inferences	B1 (1)	1.1b
(b)	[Sight or correct use of] $X \sim B(36, 0.08)$	M1	3.3
(i)	$P(X = 4) = 0.167387\ldots$ awrt **0.167**	A1	1.1b
(ii)	$[P(X \ldots 7) = 1 - P(X \text{„} 6) =]$ 0.022233… awrt **0.0222**	A1 (3)	1.1b
(c)	P(In dance club and dance tango) = 0.4×0.08 = **0.032** or $\frac{4}{125}$ or **3.2%**	B1 (1)	1.1b
(d)	[Let T = those who can dance the Tango. Sight or use of] $T \sim B(50, \text{“}0.032\text{”})$	M1	3.3
	$[P(T < 3) = P(T \text{„} 2) =]$ 0.7850815… awrt **0.785**	A1 (2)	1.1b
		(7 marks)	

Notes

(a) B1 for a suitable disadvantage:

Allow (B1)	Do NOT allow (B0)
Not random or less random (o.e.)	Not representative
Cannot use (reliably) for inferences	Less accurate
(More likely to be) biased	Any comment based on time or cost
	Any mention of skew
	Any mention of non-response

(b) M1 for sight of B(36, 0.08) Allow in words: binomial with $n = 36$ and $p = 0.08$
may be implied by one correct answer to 2sf or sight of $P(X \text{„} 6) = 0.97776\ldots$ i.e. awrt 0.98
Allow for $36C4 \times 0.08^4 \times 0.92^{32}$ as this is "correct use"

(i) 1st A1 for awrt 0.167 NB An answer of just awrt 0.167 scores M1(⇒)1st A1

(ii) 2nd A1 for awrt 0.0222

(c) B1 for 0.032 o.e. (Can allow for sight of 0.4×0.08)

(d) M1 for sight of B(50, “0.032”) ft their answer to (c) provided it is a probability ≠ 0.08
may be implied by correct answer
or sight of $[P(T \text{„} 3)] = 0.924348\ldots$ i.e. awrt 0.924 or $P(T \text{„} 2)$ as part of $1 - P(T \text{„} 2)$ calc.
A1 for awrt 0.785

MR Allow MR of 50 (e.g. 30) provided clearly attempting $P(T \text{„} 2)$ and score M1A0